Chapter: An Introduction to Conditional Probability - Basic

Conditional Probability

Learning Objectives

- Know the definition of conditional probability.
- Use conditional probability to solve for probabilities in finite sample spaces.

INDEPENDENT EVENTS – Outcomes of events are not affected by other events (in other words – random events).

DEPENDENT EVENTS – The outcome of one event is affected by another event.

MUTUALLY EXCLUSIVE EVENTS – When two events cannot occur at the same time (in a single roll, rolling a 3 on a die and rolling an even number on a die are mutually exclusive).

MUTUALLY INCLUSIVE EVENTS – When two events can occur at the same time (in a single roll, rolling a 3 on a die and rolling an odd number on a die are mutually exclusive).

In the previous section we looked at probability in terms of events that are independent and dependent, mutually inclusive and mutually exclusive. Take a look in the box to your left just to recall the definitions of these terms.

The next type of event probability is called **CONDITIONAL PROBABILITY**. With conditional probability, the probability of the second event DEPENDS ON the probability of the first event.

Conditional Probability

$$P\left(B \mid A\right) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \times P\left(B \mid A\right)$$

Another way to look at the conditional probability formula is:

$$P(second \mid first) = \frac{P(first\ choice\ and\ second\ choice)}{P(first\ choice)}$$

ABC High School students are required to write an entrance test to the statistics course before beginning the course. The following table represents the data collected regarding this year's group. The numbers represent the number of students in each group.

	Studied	**Not Studied**
Passed	17	3
Not Passed	2	23

Questions

1. Discover the following probabilities:

a. P(pass and studied)

b. P(studied) and

c. P(pass/studied)

Remember when you have completed this unit you will be see this problem again to solve it.

Let's work through a few examples of conditional probability to see how the formula works.

Example 1: A bag contains green balls and yellow balls. You are going to choose two balls without replacement. If the probability of selecting a green ball and a yellow ball is $\frac{14}{39}$, what is the probability of selecting a yellow ball on the second draw, if you know that the probability of selecting a green ball on the first draw is $\frac{4}{9}$.

Solution:

Step 1: List what you know

$$P(Green) = \frac{4}{9}$$
$$P(Green\ AND\ Yellow) = \frac{14}{39}$$

Step 2: Calculate the probability of selecting a yellow ball on the second draw with a green ball on the first draw

$$P(Y|G) = \frac{P(Green\ AND\ Yellow)}{P(Green)}$$
$$P(Y|G) = \frac{\frac{14}{39}}{\frac{4}{9}}$$
$$P(Y|G) = \frac{14}{39} \times \frac{9}{4}$$
$$P(Y|G) = \frac{126}{156}$$
$$P(Y|G) = \frac{21}{26}$$

Step 3: Write your conclusion: Therefore the probability of selecting a yellow ball on the second draw after drawing a green ball on the first draw is $\frac{21}{26}$.

Example 2: Music and Math are said to be two subjects that are closely related in the way the students think as they learn. At the local high school, the probability that a student takes math and music is 0.25. The probability that a student is taking math is 0.85. What is the probability that a student that is in music is also choosing math?

Solution:

Step 1: List what you know

$$P(Math) = 0.85$$
$$P(Math\ AND\ Music) = 0.25$$

Step 2: Calculate the probability of choosing music as a second course when math is chosen as a first course.

$$P(Music|Math) = \frac{P(Math\ AND\ Music)}{P(Math)}$$
$$P(Music|Math) = \frac{0.25}{0.85}$$
$$P(Music|Math) = 0.29$$
$$P(Music|Math) = 29\%$$

Step 3: Write your conclusion: Therefore, the probability of selecting music as a second course when math is chosen as a first course is 29%.

Example 3: The probability that it is Friday and that a student is absent is 0.05. Since there are 5 school days in a week, the probability that it is Friday is $\frac{1}{5}$ or 0.2. What is the probability that a student is absent given that today is Friday?

Solution:

Step 1: List what you know

$$P(Friday) = 0.20$$
$$P(Friday\ AND\ Absent) = 0.05$$

Step 2: Calculate the probability of being absent from school as a second choice when Friday is chosen as a first choice.

$$P(Absent|Friday) = \frac{P(Friday\ AND\ Absent)}{P(Friday)}$$
$$P(Absent|Friday) = \frac{0.05}{0.20}$$
$$P(Absent|Friday) = 0.25$$
$$P(Absent|Friday) = 25\%$$

Step 3: Write your conclusion: Therefore the probability of being absent from school as a second choice when the day, Friday, is chosen as a first choice is 25%.

Example 4: Students were asked to use computer simulations to help them in their studying of mathematics. After a trial period, the students were surveyed to see if the technology helped them study or did not. A control group was not allowed to use technology. They used a textbook only. The following table represents the data collected regarding this group. The numbers represent the number of students in each group.

	Technology	**Textbooks**
Improved studying	25	2
Did not improve studying	3	30

Discover the following probabilities:

a. P(Improved studying and used technology)

b. P(Improved studying and

c. P(Improved studying/used technology)

Solution:

Total students $= 25 + 2 + 3 + 30 = 60$

a. P(Improved studying and used technology) $= \frac{25}{60}$

P(Improved studying and used technology) $= \frac{25}{60}$

b. P(Improved studying) $= \frac{25}{60} + \frac{2}{60}$

P(Improved studying) $= \frac{27}{60}$

c. $P(Improved\ studying|used\,technology) = \frac{P(used\ technology\ \ AND\ improved\ \ studying)}{P(used\ \ technology)}$

$$P(Improved\ studying|used\ technology) = \frac{\frac{25}{60}}{\frac{28}{60}}$$
$$P(Improved\ studying|used\ technology) = \frac{25}{60} \times \frac{60}{28}$$
$$P(Improved\ studying|used\ technology) = \frac{25}{28}$$
$$P(Improved\ studying|used\ technology) = 89\%$$

Therefore the probability of improving studying when choosing technology was 89%.

Now let’s go back to our original problem from the beginning of this chapter.

ABC High School students are required to write an entrance test to the statistics course before beginning the course. The following table represents the data collected regarding this year’s group. The numbers represent the number of students in each group.

	Studied	Not Studied
Passed	17	3
Not Passed	2	23

Questions

2. Discover the following probabilities:

a. P(pass and studied)

b. P(studied, and

c. P(pass/studied)

Solution:

Total students $= 17 + 3 + 2 + 23 = 45$

a. P(passed and studied) $= \frac{17}{45}$

P(Improved studying and used technology) $= \frac{25}{60}$

b. P(studied) $= \frac{17}{45} + \frac{2}{45}$

P(studied) $= \frac{19}{45}$

c. $P(passed|studied) = \frac{P(studied\ \ AND\ \ passed)}{P(studied)}$

$$P(passed|studied) = \frac{\frac{17}{45}}{\frac{19}{45}}$$
$$P(passed|studied) = \frac{17}{45} \times \frac{45}{19}$$
$$P(passed|studied) = \frac{17}{19}$$
$$P(passed|studied) = 89\%$$

Therefore the probability of passing the course when studying was 89%.

Lesson Summary

The lesson was an extension of the previous chapter on probability. Here we learned about conditional probability or probability of events where the probability of the second occurrence is dependent on the probability of the first event. In other words, it is a probability calculation where conditions have been into place. No longer can you simply pick cards and find the probability, for example, you will now be told that the choosing of the cards have conditions. Conditions such as the first card must be a heart.

Points to Consider

- How is the conditional formula related to the previous probability formulas learned?
- Are tables a good way to visualize probability?

Vocabulary

Conditional Probability
: The probability of a particular dependent event, given the outcome of the event on which it depends.

Review Questions

1. A card is chosen at random. What is the probability that the card is black and is a 7?
2. A card is chosen at random. What is the probability that the card is red and is a jack of spades?
3. A bag contains 5 blue balls and 3 pink balls. Two balls are chosen at random and not replaced. What is the probability of choosing a blue ball after choosing a pink ball?
4. Kaj is tossing two coins. What is the probability that he will toss 2 tails given that the first toss was a tail?
5. A bag contains blue balls and red balls. You are going to choose two balls without replacement. If the probability of selecting a blue ball and a red ball is $\frac{13}{42}$, what is the probability of selecting a red ball on the second draw, if you know that the probability of selecting a blue ball on the first draw is $\frac{7}{13}$.
6. In a recent survey, 100 students were asked to see whether they would prefer to drive to school or bike. The following data was collected.

	Drive	**Bike**
Male	28	14
Female	18	40

(a) Find the probability that the person surveyed would want to drive, given that they are female.

(b) Find the probability that the person surveyed would be male, given that they would want to bike to school.

7. The little league baseball team is open to both boys and girls. The probability that a person joining the little league team and being a girl is 0.265. Of the 386 possible youth in the town to play little league ball, only 157 are girls, or 40.7%. What is the probability that a youth joining the league will be a girl?

Review Answers

1. $\frac{1}{13}$
2. 0
3. $\frac{5}{7}$
4. $\frac{1}{3}$
5. $\frac{169}{294}$
    1. $\frac{9}{29}$
    2. $\frac{7}{27}$
7. $\frac{265}{407}$

Answer Key for Review Questions (even numbers)

2. 0

4. $\frac{1}{3}$

6. (a) $\frac{9}{29}$

(b) $\frac{7}{27}$